The Development of Fraction Ideas Among Students with Disabilities

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Fraction ideas are among the most complex mathematical concepts students encounter in their school years. It is a multifaceted construct that underpinned many numeracy activities and the development of further mathematics. Secure understanding of these ideas is a vital part of being numerate. Despite its importance, there is no known research on students with disabilities learning this concept. This paper reports a pilot study of a doctorate degree to investigate one high school special education teacher and his students with disabilities' construction of fraction ideas, some of the likely and unlikely difficulties to inform future research.

Introduction

Much has been said about the importance of equity in education and assisting all students to access powerful mathematical ideas for lifelong learning. Equity in mathematics is a multidimensional issue with many forces working against students' democratic access to powerful mathematical ideas, including race, socio-economic status, and perceived students' abilities (Tate & Rousseau, 2002). In particular, many researchers argue that ability grouping - a prominent practice especially in secondary schools, discriminates against students by race, ethnicity, social class and ability (Boaler, Wiliam, & Brown, 2005). Opponents of such practice contend that it would be difficult for students to experience rich and productive conversations in a homogenous setting. However, this idea of homogeneity is being challenged by teachers who work alongside students with disabilities.

In Queensland, there are six categories of disabilities: autistic spectrum disorder (ASD), intellectual impairment (II), speech and language impairment (SLI), visual impairment (VI), hearing impairment (HI) and physical impairment (PI). Frequently, students with disabilities have a combination of impairments, some have other associated disorder such as attention deficit hyperactivity disorder (ADHD) although ADHD itself is not recognised as impairment warrant for extra funding support. These students are not alike and have a wide range of abilities and within each disability group there is considerable heterogeneity (van Kraayenoord, Elkins, Palmer, & Rickards, 2000). Extra support, resources and learning time are often needed to ensure that they produce the desired outcomes. Merely placing these students into mixed ability grouping does not guarantee access to powerful mathematical ideas. In particular, their needs might not necessarily be met within the constraint of high school mathematics classrooms. Instead, the quality of teaching is by far the most important source of variation in student achievement, after all other sources of variation are taken into account (Alton-Lee, 2003).

Currently, Australia has a shortage of teachers who are skilled and able to teach students with disabilities (Forbes, 2007). Moreover, there is a lack of specific provision in Australian schools to develop teacher knowledge for teaching numeracy to students with either learning difficulties (Louden et al., 2000) or disabilities (van Kraayenoord & Treuen, 2000). The past two MERGA 4-yearly reviews also reiterated the limited published research for this group (Forgasz et al., 2008; Perry, Anthony, &

In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2). Palmerston North, NZ: MERGA.

Diezmann, 2004). The sparsity and often dated research means that we know very little about these students' mathematics learning. Internationally, the amount of research devoted to mathematics instruction for students with disabilities, especially those with mental retardation, has remained relatively constant over the past 25 years with an average of 16 studies spanning a 10-year period (Butler, Miller, Lee, & Pierce, 2001). In particular, many studies place substantial emphasis on using experimental design to investigate the effect of various instructions such as direct instruction, drill and practice and time delay on basic skill training and computation. This exclusive focus on rules and algorithm has unavoidably prevented many students with disabilities from experiencing rich mathematical ideas.

There are three assumptions that may have produced such insipid and often predictable research. First is the stereotypical belief that these students are incapable of devising cognitive strategies and engaging in mathematical problem solving. Secondly, the belief that mathematics is sequential suggests a need to fully teach each concept in succession. Third is the lack of collaboration and communication among special education and mathematics researchers and classroom teachers. Consequently, valuable resources are wasted in conducting research intervention that produces minimum influence on classroom practice. Schooling for most students, those with disabilities in particular, "is an endless sequence of memorising and forgetting facts and procedures that make little sense to them (Battista, 1999, p. 426). Battista warned that such teaching would seriously stunt the growth of students' mathematical reasoning and problem-solving skills. Indeed, "in the information age and the web era, obtaining facts is no longer the problem; analysing and making sense of them is" (Battista, 1999, p. 428).

Accordingly, numeracy development for students with disabilities should be given greater emphasis when making curriculum decisions. Numeracy is the ability to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Australian Association of Mathematics Teachers (AAMT), 1997, p. 15). It is contextualised and requires flexible thinking and using of mathematics to solve problems. Since much of the daily demands in life involves proportional reasoning, such as perusing a recipe, home loans, interest rates, phone deals and other civic related activity, it follows that students with disabilities must be taught how to handle rational number as well.

Rational numbers is considered to be among the most difficult topics to learn in foundational mathematics (McGee, Kervin, & Chinnappan, 2006). The multifaceted nature of the concept means that careful attention must be given to develop the meanings of different rational number representations and the connections among them. Despite the voluminous amount of literature produced, how students become proficient with rational numbers is not as well understood as with whole numbers (Kilpatrick, Swafford, & Findell, 2001). In particular, earlier experiences often cause serious conflicts when learning rational numbers, such that $^1/_9$ is considered as bigger than $^1/_8$ and $^3/_5 + ^2/_4 = ^5/_9$. To date, there is no known research study that looks at how students with disabilities learn rational number. Consequently, this preliminary study seeks to survey students with disabilities' initial fraction ideas and investigate the effectiveness of using instructional games to help students connect different representations to the appropriate fractions. The aim is to investigate one special education teacher and his class's construction of fraction ideas and use the knowledge gained to inform future investigation.

Method

This paper takes the position that sustained transformation of practice occurs when research is conducted with teacher and students as they construct mathematical knowledge together. It uses the principles of participatory action research (McTaggart, 1997) to conduct a series of flexible cycles of action and reflection between the researcher and the teacher as they examine the effect of each learning task on students' understanding of initial fraction ideas. In particular, it focuses on students' understanding of equal and unequal parts, number lines and partitioning. The strategies and instructional games presented here draw on the work of George Booker (Booker, 2000; Booker et al, 2004). The initial introduction focuses on helping students construct mental representations of fraction ideas and the language associated with them (see figure 1) through games and hands on activities involving fraction models and number lines.

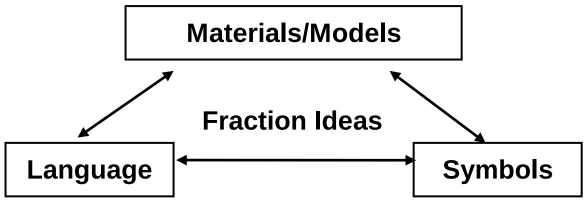

Figure 2: Booker's connected sequence of mathematical development (Booker et al., 2004)

Developing secure understanding of the part/whole relationship is crucial because struggling learners' knowledge about fractions tend to be rigid and fragmented, focusing solely on "rules" and procedures rather than knowing why and finding creative ways of solving problems. Consequently, multiple representations are used to link concrete materials and models with appropriate corresponding ordinal names to investigate whether students are able to comprehend fraction ideas.

Participants and Data Collection

This study is conducted in a Brisbane high school situated in a low socio-economic area. The school's Special Education Program (SEP) runs a modified program for the four main key learning areas (English/Math/SOSE/Science) for Year 8 to Year 10 students with disabilities. The special education teacher Terry and his Year 8 class were selected to participate in this action research. Despite the lack of formal training in special education and mathematics, Terry has had extensive experience working with students with disabilities and other educationally disadvantaged students. He is enthusiastic and views rational numbers as an important construct his students need for lifelong learning. His class consists of Tim who has ASD, Jack who has learning difficulties, May, and Cassie have speech and language impairment and five students, Pete, Kerry, Sale, Jacob and Ashley, with intellectual impairment. An initial interview was carried out with the whole class by the researcher to identify students' level of understanding in relation to fraction ideas. Information collected is then used to initiate the cycle of planning, teaching, and evaluating process (Kemmis & Wilkinson, 1998). Instructions were modified and presented repeatedly to ensure that each student understand the task demand. Students were presented with a series of questions such as:

1. Have you heard of the word "fractions"?

2. Can you draw or write something about “fractions” and show me how it looks like?

Students were then given a task sheet and asked to shade each circle according to the instructed parts given by the researcher. The questions were:

1. Can you cut the circle into half?
2. Can you cut the circle into three equal parts?
3. Can you cut the circle into four equal parts? Can you then shade three out of the four equal parts?
4. Can you cut the circle into five equal parts? Can you then shade two out of the five equal parts?
5. Can you cut the circle into six equal parts? Can you then shade three out of the six equal parts?
6. Can you cut the circle into eight equal parts? Can you then shade all the eight equal parts?

This is followed by a series of teaching activities designed to develop students’ understanding of initial fraction ideas. The intervention took place three times per week during their numeracy block. Each session lasted between 30 to 40 minutes depending on the task demand and students’ responsiveness. Weekly discussion between the researcher and Terry involves evaluation of previous lessons, assessment on students’ response to task, and planning for next stage of students’ learning. The following reports the result of one term’s intervention.

Results and Discussion

Seven students were present on the day of the baseline interview. Unfortunately, Tim was very uncooperative on that day and refused to participate. It could be that the change of routine has upset his need for consistency and thus he was unable to deal with the change. Students had great difficulty drawing or writing something to represent fractions. Jack was the only one who wrote: ½ + ¾ =. When asked how he would say what he wrote, he replied “one two plus three four equal. Since Jack has learning difficulty, it can be assumed that he might have came across this form of fraction ideas in regular class. Nevertheless, his fragmented knowledge is revealed through his inability to address the fractions appropriately.

The results for the second task were presented below:

Table 1.

Number of Correct Response N=7

½	$^1/_3$	¾	$^2/_5$	$^3/_6$	$^8/_8$
6	0	4	0	4	4

On the whole, these students’ knowledge about fraction ideas was sketchy and restricted to their daily experience. Half is a concept that all of them would have encountered in their daily life so it was not difficult to perform. 1 third and 2 fifths present great challenge for these students, as none was able to cut it into equal parts. In fact, two students consistently failed to segment the parts equally. Four students have reasonable success with 3 fourth and 8 eights. The result for 3 sixths was inconclusive as none of them were able to segment the parts equally but all four students managed to shade half of the circle. It could be that a lack of spatial awareness has prevented students from segmenting the parts equally.

The intervention took place in Term 4 of year 8. Many of these students do not have much prior knowledge about fraction ideas. They also do not have experience working in groups and can give up easily or disrupt the class as soon as the task becomes too hard. A list of instructional games and concrete materials such as fractions pieces were used to build students' interest and develop students' understanding of equal parts. The aim is to link the language and symbol with the appropriate representations (see figure 2). Kroesbergen and Van Luit (2003) assert that leaving students especially those with learning difficulties and disabilities, to discover hidden knowledge in the resource materials alone is unlike to succeed. In each lesson, explicit instructions were given to help students recognise, order and compare fractions. Instructions were repeated and rephrased when necessary to make sure that students comprehend the task demand.

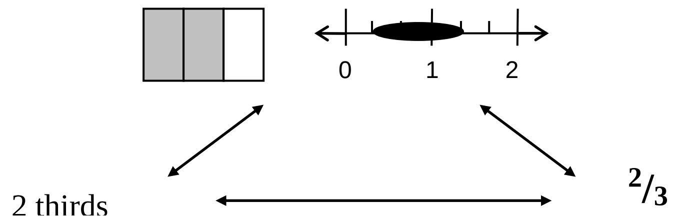

Figure 2: Instructions to link representations and language with the appropriate symbol.

Both the *Colour a Fraction Game* (Booker, 2000) and identifying "*Equal and Unequal Parts*" were met with reasonable success although explicit instructions had to be given to ensure that students can identify the difference between each representation and get used to the language used to describe each fraction model. Students were more confident in dealing with fraction models than with number lines. It is possible that these students are more familiar with the idea of fraction models than the concept of fraction as a point on a number line. A number of researchers have also noted how knowledge of whole number operations can interfere with students' development of fraction ideas (Behr & Post, 1988; Pearn & Stephens, 2007). Indeed, these students were getting quicker at counting spaces on the number line to obtain the correct answer. However, a shift of thinking based on a counting algorithm is needed if their knowledge about fraction ideas were to progress further (Behr & Post, 1988). Three students continue to face difficulty in matching fraction models or number line with the appropriate words, due largely to their lower reading ability.

The paper folding activity was met with much frustration. One explanation could be that many of them do not have well developed fine motor control and thus has unavoidable affected their performance on the task. Consequently, many were annoyed by the crumpled strip of paper. The other reason could also due to a lack of spatial awareness. However, we not yet been able to design a task to check on this development.

The last activity required students to fold fraction circles and strips of papers into half and quarters and then name which of the two is larger. Although students were able to carry out the task with reasonable success, none of them could correctly identify $^{1}/_{7}$ as smaller than ¼ when the question was given straight after the folding activity. It appears that how a task is presented and the timing of its introduction can significantly impact students' construction of mathematical ideas. In this case, the comparing task was introduced too quickly without knowing whether students had developed an understanding of the part-whole relationship. Indeed, further investigation is needed to consider this aspect of the learning process.

Discussion with Terry revealed a greater insight into the difficulty teachers encounter and a need for professional development to focus on developing teachers' mathematical content knowledge, pedagogical knowledge, knowledge about their students' learning and the relationships among them. He noted that the weekly discussions with the researcher alerted him to pay attention to how individual students construct mathematical ideas and assisted in his planning process. He was able to target a specific area of difficulties and work out a strategy to help individuals construct their own knowledge. The structured, sequential step-by-step instructions were very beneficial for his students as it helps to draw their attention to each component without feeling overwhelmed by the amount of information at hand. The variety of visual and hands on activities created high interest and students were very engaged. Since the starting of this project, his students are beginning to enjoy working in groups whereas previously they were intolerable towards each other. This is particularly significant for Tim who struggles with group work and changes of routine. Terry highlighted that the language of mathematics played an important part in his students' understanding of fraction ideas. He asserted that the process has helped him learn to give instruction that is concise and straight to the point. He pointed out that the language used in the past (e.g. 1 over 4 rather than 1 out of 4) continues to influence the way some students operate. Nevertheless, he believes that in time to come this problem will be overcome so long as students continue to be immersed in the rich and appropriate mathematical conversations. He was quite excited by the fact that one of his students was able to link the vinculum to the dividing symbol (÷).

Moreover, Terry commented that not knowing the right starting point to teach is the biggest frustration he faced. He lamented that:

> I look through all the resource books I can find but they don't tell you if it's for which level. I've no way of knowing if the activity is too hard or too easy for my students and I don't know what to do after that. The pictures in the textbook are two dimensions and these students need concrete materials that they can touch and play with.

The weekly discussions have helped Terry better understand the difficulties students faced, broaden his teaching repertoire and empowered him to construct knowledge of his own. He repeatedly emphasised the need for activities that provide sequential step-by-step instructions so that teachers are better able to handle uncertainty in class. He maintained that this series of developing students' initial fraction ideas has help to build the basis where he could carry his students through. "Whenever they are stuck, we always go back to the basic and try again. For these students this is important".

Conclusion and Implications

This study set out to survey students with disabilities' knowledge about fraction ideas and the type of instructions that best promote learning. The data collected have help to construct a series of questions for future research. Firstly, the idea of explicit instruction based on constructivism is not well understood. Within the special education literature, explicit instruction often denotes direct instruction and drill and practice. Further research is needed to assist teachers in framing their instructions to assist students to think mathematically.

Relating to the issue of instruction is the need for a structured program to help teachers of non-mathematics teach rational numbers. Currently, there is no known program that teaches students with disabilities to learn fractions. Timing and pacing in instructional delivery certainly play a huge part in the success of students' learning. Moreover, we do not know to what extent the impact of a disability has on a student's construction of mathematical ideas. It is clear that more research is needed in the area of professional development to link these issues together to improve current practice.

Instructional games and concrete materials were found to be very effective in developing students' engagement and initial fraction ideas. Unfortunately, the timeframe for this study was too short to truly evaluate each task and how they could be linked to assist students' understanding. Consequently, this may have affected students' performance.

The results from the initial baseline interview revealed that many students with disabilities have very limited understanding of what fraction is and how that relates to their daily activity. If they are to become numerate, they must be given the chance to experience different mathematical ideas. Only by doing so could the true meaning of equity in education be realised.

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